Matrix Multiplication Made Easy: 4 Visual Ways to Master the Math

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November 15, 2025

1 Introduction

Matrix multiplication is the workhorse of the modern world. From powering the deep learning algorithms for large language models to rendering the photo-realistic graphics in video games – it's everywhere.

A solid understanding of this process is fundamental to anyone working with data. Unfortunately, many of us are taught just one rigid way to perform matrix multiplication—the traditional "row-times-column" method—and that's often where the learning stops. Seeing that matrix multiplication can be approached in different ways is a real revelation that will boost your problem-solving and conceptual understanding skills.

In this guide, I'll present four distinct ways to perform matrix multiplication and I'll illustrate them with clear, visual explanations, making it easy to internalize the concepts. While the core ideas about these varied perspectives can be found in excellent resources like [1], the description in many texts is often very short. My contribution is to provide a gentle, step-by-step approach, coupled with dedicated visualizations for each method, along with clear numerical examples.[1]

First, let's quickly establish some common notations I'll use throughout this guide:

- Matrix: A matrix will be denoted as an uppercase bold letter, e.g. $\mathbf{A}$ denotes a matrix.
- Columns: I'll denote the i-th column of a matrix using a lowercase bold letter with a subscript. For example, $\mathbf{a}_i$ will represent the i-th column of matrix A.
- Dimensions: The variable m will always refer to the number of rows in a matrix, and n will denote the number of columns. So, if we say $\mathbf{A} \in \mathbb{R}^{3\times4}$, it means matrix $\mathbf{A}$ has $m = 3$ rows and $n = 4$ columns. (Note: We'll use $\mathbb{R}^{m\times n}$ for clarity in the blog post rather than $\mathbb{R}^{m,n}$).
- Multiplication Compatibility: The product of two matrices, $\mathbf{A}$ and $\mathbf{B}$, can only be computed if the number of columns in $\mathbf{A}$ exactly matches the number of rows in $\mathbf{B}$. If $\mathbf{A}$ is an $m \times n$ matrix and $\mathbf{B}$ is an $n \times p$ matrix, then the resulting product $\mathbf{AB}$ will be a new matrix with dimensions $m \times p$ (number of rows of $\mathbf{A}$ by number of columns of $\mathbf{B}$).Visually, you can think of it like this: if $\mathbf{A} \in \mathbb{R}^{m\times \mathbf{n}}$ and $\mathbf{B} \in \mathbb{R}^{\mathbf{n}\times p}$, the "inner" dimensions (n) must match, and the "outer" dimensions (m and p) define the size of the result.

2 Perspective 1: Columns of A, Weighted by Entries of b

Let's begin with the basics: what happens when we multiply a matrix $\mathbf{A}$ by a matrix that is just a single column vector $\mathbf{b}$? This perspective is very intuitive and forms the basis for understanding more complex matrix multiplications. (When $\mathbf{B}$ is a single column vector meaning $\mathbf{B} \in \mathbb{R}^{n\times1}$ we can also refer to it as $\mathbf{b}$.) The product $\mathbf{AB}$ results in a new column vector. This result is a linear combination of the columns of $\mathbf{A}$, where each column of $\mathbf{A}$ is 'weighted' (multiplied) by the corresponding entry from $\mathbf{b}$. In simpler terms: If $\mathbf{A}$ has columns $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ and $\mathbf{b}$ is a column vector with entries $b_1, b_2, \ldots, b_n$, then:

$$A\mathbf{b} = b_1\mathbf{a}_1 + b_2\mathbf{a}_2 + \cdots + b_n\mathbf{a}_n \tag{1}$$

This means the product $\mathbf{Ab}$ is simply the sum of each column of $\mathbf{A}$, scaled by the respective entry in $\mathbf{b}$. This relationship is clearly visualized in Figure 1, where you can see how each

[1] I've used AI for proofreading to ensure clarity as English is not my first language.

column of **A** is ”stretched” or ”shrunk” by an element of **b** before being combined.

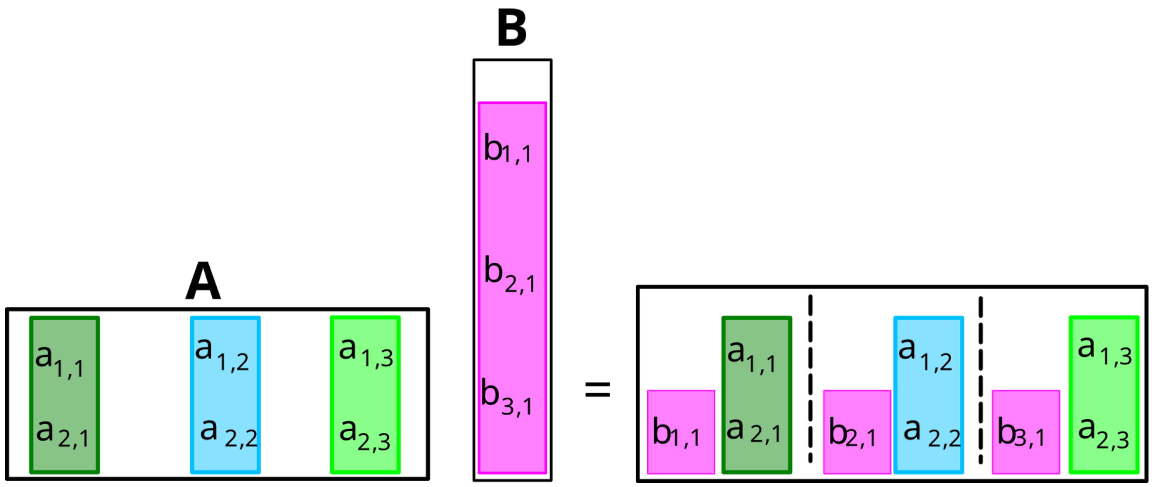

Figure 1: Visualizing $\mathbf{A} \times \mathbf{B}$ when $\mathbf{B}$ is a single column. The product shows each column of $\mathbf{A}$ being weighted (scaled) by its corresponding element in $\mathbf{B}$, resulting in a linear combination of $\mathbf{A}$'s columns.

2.1 Numerical Example

Let's consider a numerical example for $\mathbf{A} \times \mathbf{b}$:

Let matrix $\mathbf{A}$ be:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

And let the column vector $\mathbf{b}$ be:

$$\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Following Perspective 1, we treat $\mathbf{b}$'s entries as weights for $\mathbf{A}$'s columns. The columns of $\mathbf{A}$ are: $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $\mathbf{a}_3 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$.

Then, the product **Ab** is:

$$\begin{aligned}\mathbf{Ab} &= b_1\mathbf{a}_1 + b_2\mathbf{a}_2 + b_3\mathbf{a}_3 \\ &= 2\begin{bmatrix}1\\4\end{bmatrix} + 1\begin{bmatrix}2\\5\end{bmatrix} + 3\begin{bmatrix}3\\6\end{bmatrix} \\ &= \begin{bmatrix}2\\8\end{bmatrix} + \begin{bmatrix}2\\5\end{bmatrix} + \begin{bmatrix}9\\18\end{bmatrix} \\ &= \begin{bmatrix}2+2+9\\8+5+18\end{bmatrix} \\ &= \begin{bmatrix}13\\31\end{bmatrix}\end{aligned}$$

This clearly shows how each column of **A** is scaled by the corresponding entry from **b** before they are summed to form the final result.

3 Perspective 2: Rows × Columns Dot Products

The second perspective is the most common—it's likely the method you learned first in school or university. This approach focuses on calculating each individual entry in the resulting matrix, $\mathbf{C} = \mathbf{AB}$. In this view, every entry $c_{i,j}$ (the element in row i and column j of $\mathbf{C}$) is computed as a dot product between the i-th row of $\mathbf{A}$ and the j-th column of $\mathbf{B}$.

Recall the Dot Product: The dot product of two column vectors **a** and **b** of the same length r is a single scalar number.

$$\mathbf{a}^T\mathbf{b} = a_1b_1 + a_2b_2 \ldots + a_rb_r = \sum_{i=1}^{r} a_ib_i \tag{2}$$

(The superscript T denotes the transpose, which means the column vector turns into a row vector. This is necessary for the dimensions to match.)

Here is a numerical example for two vectors $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [4, 5, 6]$:

$$\mathbf{a}^T\mathbf{b} = \begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}4\\5\\6\end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6 = 32 \tag{3}$$

Thus, when you multiply two matrices, each entry in the resulting matrix $c_{i,j}$ is defined as:

$$\text{Entry } c_{i,j} = (\text{Row } i \text{ of } \mathbf{A}) \cdot (\text{Column } j \text{ of } \mathbf{B}) \tag{4}$$

This process is visualized in Figure 2, highlighting how the matching row and column "meet" to produce a single number in the result matrix.

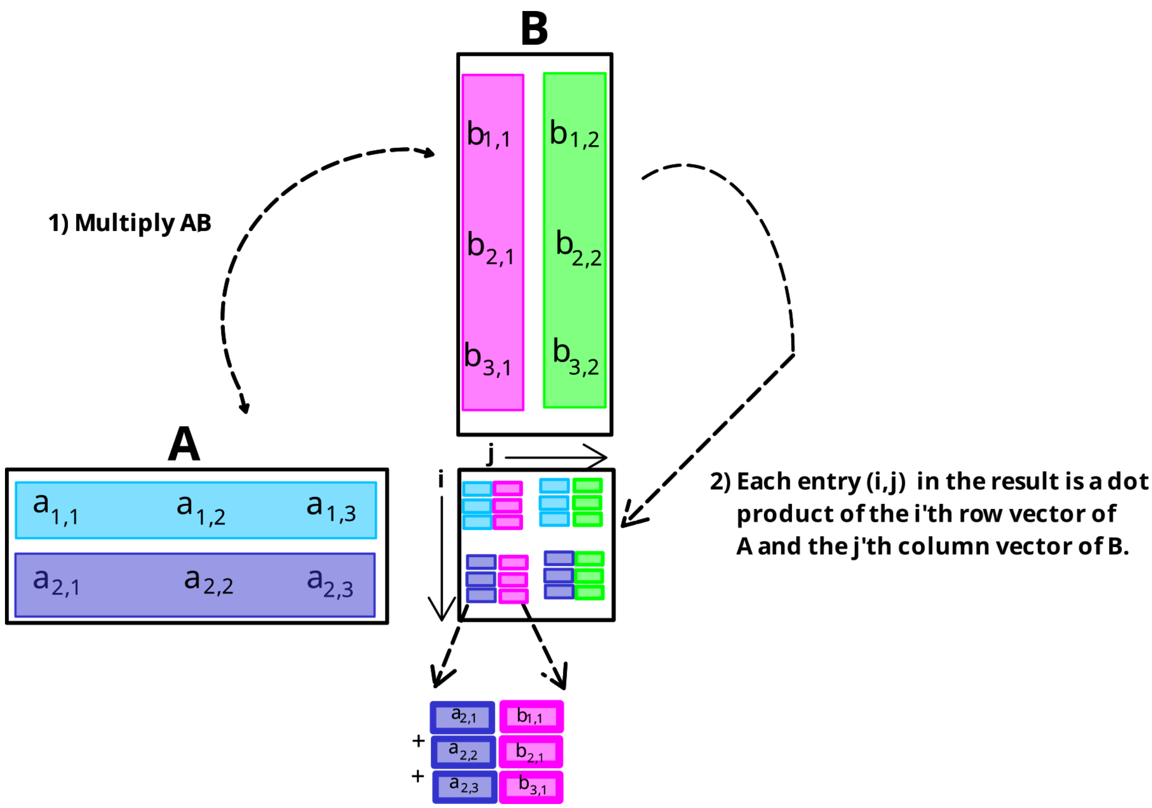

Figure 2: In this figure, matrix **A** is displayed on the left, and matrix **B** is positioned on top. The resulting product **AB** appears in the center, visually aligning with the rows of **A** and the columns of **B**. This arrangement clearly illustrates which rows of **A** are multiplied by which columns of **B** to form the entries of the product matrix. Specifically, the product **AB** contains the dot product of row i of **A** and column j of **B** at position (i, j). This relationship is explicitly demonstrated in the figure for position $(2, 1)$.

3.1 Numerical Example

Let's illustrate the "Rows $\times$ Columns Dot Products" perspective with a numerical example.

Consider two matrices **A** and **B**:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 7 & 8 \\ 9 & 1 \\ 2 & 3 \end{bmatrix}$$

To find the product $\mathbf{C} = \mathbf{AB}$, we compute each entry $c_{i,j}$ by taking the dot product of the i-th row of $\mathbf{A}$ and the j-th column of $\mathbf{B}$. The resulting matrix $\mathbf{C}$ will be 2×2.

Let's calculate each entry:

1. **For $c_{1,1}$ (Row 1 of A $\cdot$ Column 1 of B):**

$$c_{1,1} = [1, 2, 3] \cdot \begin{bmatrix} 7 \\ 9 \\ 2 \end{bmatrix} = (1 \times 7) + (2 \times 9) + (3 \times 2) = 7 + 18 + 6 = 31$$

2. **For $c_{1,2}$ (Row 1 of A $\cdot$ Column 2 of B):**

$$c_{1,2} = [1, 2, 3] \cdot \begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix} = (1 \times 8) + (2 \times 1) + (3 \times 3) = 8 + 2 + 9 = 19$$

3. **For $c_{2,1}$ (Row 2 of A $\cdot$ Column 1 of B):**

$$c_{2,1} = [4, 5, 6] \cdot \begin{bmatrix} 7 \\ 9 \\ 2 \end{bmatrix} = (4 \times 7) + (5 \times 9) + (6 \times 2) = 28 + 45 + 12 = 85$$

4. **For $c_{2,2}$ (Row 2 of A $\cdot$ Column 2 of B):**

$$c_{2,2} = [4, 5, 6] \cdot \begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix} = (4 \times 8) + (5 \times 1) + (6 \times 3) = 32 + 5 + 18 = 55$$

Therefore, the product matrix $\mathbf{C}$ is:

$$\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 31 & 19 \\ 85 & 55 \end{bmatrix}$$

4 Perspective 3: Matrix $\times$ Columns (Column-by-Column Transformation)

This third perspective builds directly upon our first one, where we multiplied a matrix by a single column vector. Here, we recognize that the overall matrix multiplication $\mathbf{AB}$ can be broken down into a series of such individual operations. Specifically, each column j in the resulting product matrix $\mathbf{C} = \mathbf{AB}$ is obtained by multiplying the first matrix $\mathbf{A}$ by the j-th column of the second matrix $\mathbf{B}$. So, if $\mathbf{B}$ has columns $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p$, then the product $\mathbf{AB}$ will have columns $\mathbf{Ab}_1, \mathbf{Ab}_2, \ldots, \mathbf{Ab}_p$:

$$\mathbf{AB} = \mathbf{A}\begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2 & \ldots & \mathbf{b}_p\end{bmatrix} = \begin{bmatrix}\mathbf{Ab}_1 & \mathbf{Ab}_2 & \ldots & \mathbf{Ab}_p\end{bmatrix} \tag{5}$$

This perspective is incredibly useful because it highlights how matrix $\mathbf{A}$ transforms each column vector of $\mathbf{B}$ independently. Each column of the result is a linear combination of the columns of $\mathbf{A}$, with the weights coming from the corresponding column of $\mathbf{B}$. Refer to Figure 3 for a clear visualization of this column-by-column transformation.

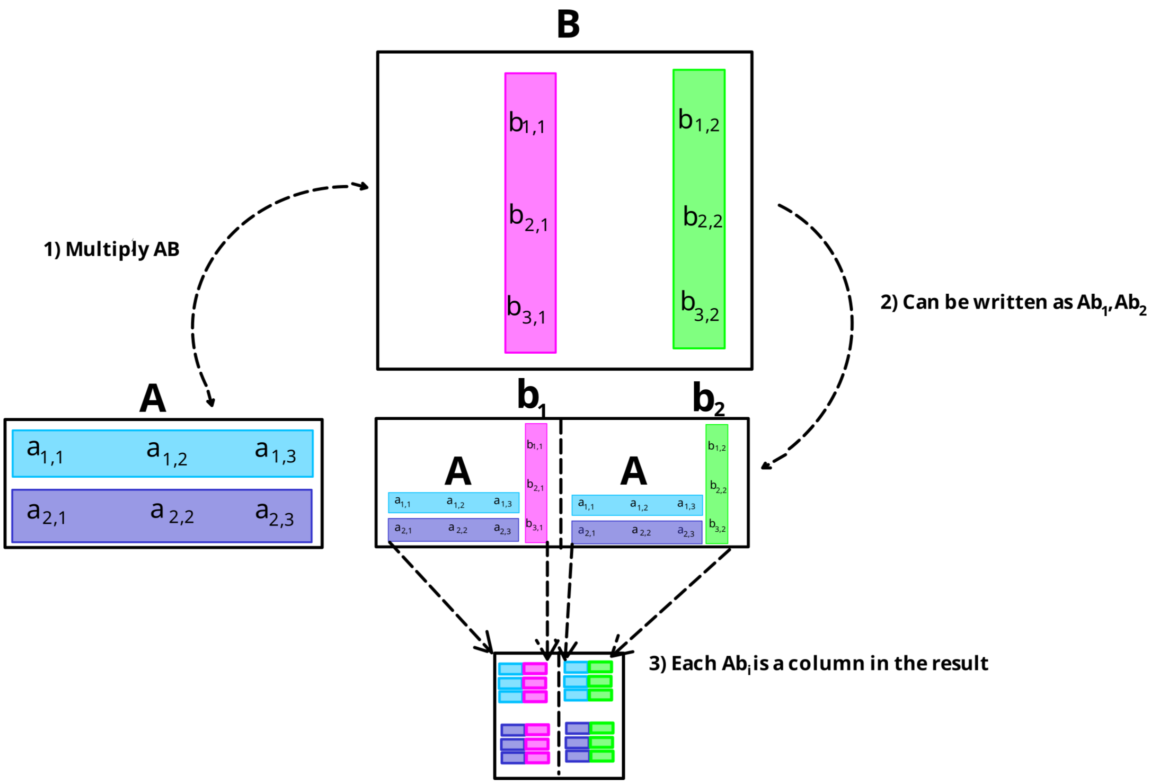

Figure 3: Again, $\mathbf{A}$ is shown on the left and $\mathbf{B}$ is positioned at the top, with $\mathbf{AB}$ is displayed in the center. Each column j in the result represents a matrix-vector multiplication, where $\mathbf{A}$ is multiplied by the j'th column of $\mathbf{B}$.

Since we've already demonstrated a numerical example of matrix-vector multiplication in Perspective 1, I'll forego another one here. The core calculation for each $\mathbf{Ab}_j$ is identical to what we saw previously.

5 Perspective 4: Columns × Rows (Summing Outer Products)

The fourth and final perspective we'll explore is somehow the opposite of the "dot product view," as it involves using outer products. This method emphasizes how the product matrix **AB** can be constructed by summing up a series of smaller matrices.

Recall the Outer Product: An outer product is formed by multiplying a column vector by a row vector, and the result is a matrix. This is distinct from a dot product, which multiplies a row by a column and results in a single scalar number. If $\mathbf{u}$ is an $m \times 1$ column vector and $\mathbf{v}^T$ is a $1 \times p$ row vector, their outer product $\mathbf{u}\mathbf{v}^T$ is an $m \times p$ matrix. When we multiply two matrices, $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times p}$, we can compute $\mathbf{AB}$ by summing up a series of outer products. Specifically, we take each column of $\mathbf{A}$ and multiply it by the corresponding row of $\mathbf{B}$. Let $\mathbf{a}_k$ denote the k-th column of $\mathbf{A}$ and $\mathbf{b}_k^T$ denote the k-th row of the transposed $\mathbf{B}$. (The notation might be confusing, it is simply the k'th row of $\mathbf{B}$ but $\mathbf{b}_k^T$ looks as if it was the first column of $\mathbf{B}$ and then transposed, but that's not what it is. It is the first column of the transposed $\mathbf{B}$, and the transposed again. This is just a way to denote a row of a matrix.) The product $\mathbf{AB}$ is then the sum of these outer products:

$$\mathbf{AB} = \mathbf{a}_1\mathbf{b}_1^T + \mathbf{a}_2\mathbf{b}_2^T + \cdots + \mathbf{a}_n\mathbf{b}_n^T \tag{6}$$

Each term $\mathbf{a}_k\mathbf{b}_k^T$ is an $m \times p$ matrix, and their sum yields the final $m \times p$ product matrix $\mathbf{AB}$. This perspective is powerful for understanding concepts like Singular Value Decomposition (SVD) and low-rank approximations. Refer to Figure 4 for a visual demonstration of summing these outer products.

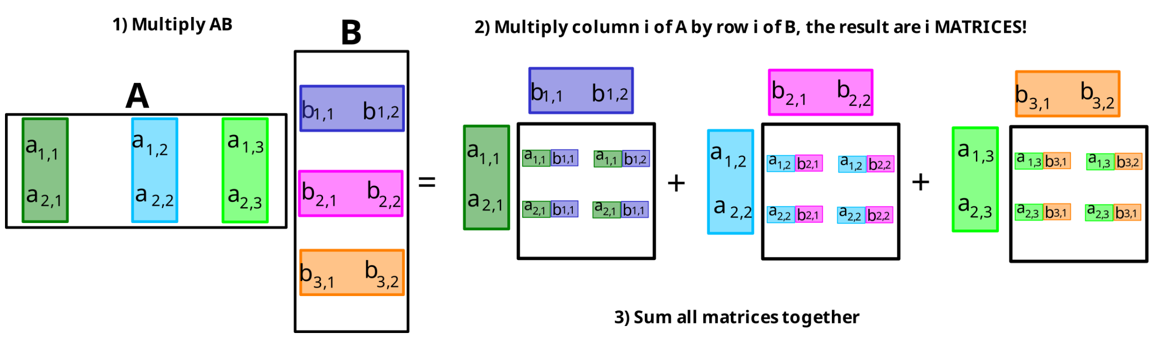

Figure 4: The product of two matrices is the sum of their outer products, meaning each column of **A** is multiplied by the corresponding row of **B**. In the figure, the column of **A** is shown on the left, while the corresponding row of **B** is displayed at the top, allowing the resulting product - a matrix - to be presented in the center.

5.1 Numerical Example

Let's use a numerical example to demonstrate the "Summing Outer Products" method.

Consider two matrices **A** and **B**:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \tag{7}$$

Here, **A** has columns $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. And **B** has rows $\mathbf{b}_1^T = [5, 6]$ and $\mathbf{b}_2^T = [7, 8]$.

According to this perspective, **AB** is the sum of outer products: $\mathbf{a}_1\mathbf{b}_1^T + \mathbf{a}_2\mathbf{b}_2^T$.

1. **First Outer Product:** $\mathbf{a}_1\mathbf{b}_1^T$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 \times 5 & 1 \times 6 \\ 3 \times 5 & 3 \times 6 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 15 & 18 \end{bmatrix}$$

2. **Second Outer Product:** $\mathbf{a}_2\mathbf{b}_2^T$

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} \begin{bmatrix} 7 & 8 \end{bmatrix} = \begin{bmatrix} 2 \times 7 & 2 \times 8 \\ 4 \times 7 & 4 \times 8 \end{bmatrix} = \begin{bmatrix} 14 & 16 \\ 28 & 32 \end{bmatrix}$$

Now, sum these two outer product matrices to get the final product **AB**:

$$\begin{aligned} \mathbf{AB} &= \begin{bmatrix} 5 & 6 \\ 15 & 18 \end{bmatrix} + \begin{bmatrix} 14 & 16 \\ 28 & 32 \end{bmatrix} \\ &= \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} \\ &= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \end{aligned}$$

This perspective is fundamental across linear algebra and its applications. It's the core idea behind concepts like Singular Value Decomposition (SVD), where a matrix is decomposed into a sum of simple outer products, allowing for powerful techniques like dimensionality reduction, low-rank approximations, and recommendation systems. Understanding this view unlocks a deeper insight into how matrices can capture and represent complex relationships.

6 Summing Up: Four Ways to View Matrix Multiplication

We've explored four distinct ways to interpret matrix multiplication, moving beyond the single "row-times-column" method. Each perspective offers unique insights:

1. Matrix-Vector Product: $\mathbf{AB}$ (where $\mathbf{B}$ is a single column) is a linear combination of $\mathbf{A}$'s columns, weighted by $\mathbf{B}$'s entries.
2. Row-by-Column Dot Products: Each entry in $\mathbf{AB}$ is the dot product of a row from $\mathbf{A}$ and a column from $\mathbf{B}$. (The traditional method).
3. Matrix-by-Columns: Each column of $\mathbf{AB}$ is $\mathbf{A}$ multiplied by the corresponding column of $\mathbf{B}$.
4. Columns-by-Rows (Sum of Outer Products): $\mathbf{AB}$ is the sum of outer products, where each is a column of $\mathbf{A}$ times a corresponding row of $\mathbf{B}$.

Congratulations for making it this far! By exploring these four distinct perspectives, you've moved beyond rote calculation and gained a deeper, more intuitive understanding of matrix multiplication.

References

[1] Gilbert Strang. *Introduction to Linear Algebra, Fifth Edition.* Wellesley-Cambridge Press, 2016.