NONLINEAR ESTIMATION USING CENTRAL DIFFERENCE INFORMATION FILTER

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ABSTRACT

In this contribution, we introduce a new state estimation filter for nonlinear estimation and sensor fusion, which we call central difference information filter (CDIF). As we know, the extended information filter (EIF) has two shortcomings: one is the limited accuracy of the Taylor series linearization method, the other is the calculation of the Jacobians. These shortcomings can be compensated by utilizing sigma point information filters (SPIFs), e.g., the unscented information filter (UIF), which uses deterministic sigma points to approximate the distribution of Gaussian random variables and does not require the calculation of Jacobians. As an alternative to the UIF, the CDIF is derived by using Stirling's interpolation to generate sigma points in the SPIFs architecture, which uses less parameters, has lower computational cost and achieves the same accuracy as UIF. To demonstrate the performance of our algorithm, a classic space vehicle reentry tracking simulation is used.

***Index Terms*—** Nonlinear estimation, multiple sensor fusion, target tracking, sigma point filters, central difference information filter.

1. INTRODUCTION

Control systems can be more accurate, complete and robust by using fused information from multiple sensors. Therefore, multiple sensor fusion techniques have been widely studied in many research fields, i.e., robot navigation, surveillance, and intelligent vehicles [1, 2, 3]. Recently, the information filter (IF), which is the dual of the Kalman filter (KF), has attracted much attention for multiple sensor fusion. Both the IF and the KF represent distribution of random state variables with Gaussians. However, in contrast to moment parametrization as done in the KF, the IF uses an information matrix and an information vector to represent the Gaussians. This difference in parameterization makes the IF superior to the KF concerning multiply sensor fusion, i.e., computations are simpler and no prior information of the system state is required [1].

In the case of nonlinear estimation problems, an extended version of the IF can be obtained by using the first order term of the Taylor series expansions of the nonlinear functions, which is called extended information filter (EIF). This approximation can introduce large errors when the system model is highly nonlinear, and the higher order terms of Taylor series are important [4]. To address this issue, the unscented information filter (UIF) has been proposed by Kim [2] and Lee [1]. Kim developed the UIF by using minimum mean square error estimation. By contrast, Lee's UIF algorithm is derived by embedding statistical linear error propagation into the EIF architecture. Although their methods are different, results are essentially identical [1] [2]. The UIF uses a number of deterministic sigma points to capture the true information matrix and information vector, which can be accurate up to the second order of any nonlinearity. However, three parameters (α, β, κ) are needed to be defined first in UIF, which depend on the system models. As shown in [1], the UIF is superior to the EIF not only in terms of estimation accuracy but also concerning the convergence speed for nonlinear estimation and multiple sensor fusion. However, the choice of system parameters (α, β, κ) can affect the filter's estimation precision.

In this paper, we employ Stirling's interpolation in the IF for nonlinear estimation and multiple sensor fusion problems. Stirling's interpolation replaces the unscented transform in the UIF algorithm architecture. As proved in [4], Stirling's interpolation based central difference Kalman filter (CDKF) has the same or superior performance as the unscented transform based unscented Kalman filter (UKF), with one advantage over UKF: the Stirling's interpolation only needs one single parameter which is the interval size h, whereas the unscented transform needs three [5]. Therefore, our motivation is combining Stirling's interpolation with the IF, which lead to the central difference information filter (CDIF). As shown in our simulation experiment, the CDIF not only inherits the simplicity of the IF for multiply sensor fusion, and has the same accuracy as the UIF, but also has lower computational cost.

The paper is organized as follows: in Section 2, Stirling's interpolation method is introduced. Section 3 presents our CDIF algorithm for nonlinear estimation and multiple sensor

This work has been supported by the EU project Garnics under contract No. 247947 and the EU project PACO-PLUS under contract No. 027657.

fusion. The simulation results of target tracking are presented and discussed in Section 4. Finally, the work is concluded in Section 5.

2. STIRLING'S INTERPOLATION

Stirling's interpolation was used with a Kalman filter in the literature for a while, which is called CDKF [5, 4]. It uses a symmetric set of $2L+1$ sigma points to approximate nonlinear functions, whereas the EKF uses the Taylor series. The advantage of the Stirling's interpolation is that the calculation of Jacobians is not required, and can be accurate up to the second order of any nonlinearity. In case of Gaussian distributions of the system variables, the mean and covariance can be represented by those sigma points. As we mentioned in Section 1, the IF is a dual filter of the KF, such that the information vector and matrix also can be derived by those sigma points. In this section, we first show how the mean and covariance are derived using Stirling's interpolation, then the information vector and matrix are obtained from the mean and covariance.

The $2L+1$ prior sigma points used in Stirling's interpolation are given by the prior mean $\hat{x}$ plus or minus the columns of the scaled square root of the prior covariance matrix P_x [4]:

$$\chi_i = \begin{cases} \hat{x}, & i=0 \\ \hat{x}+(h\sqrt{P_x})_i, & i=1,\cdots,L \\ \hat{x}-(h\sqrt{P_x})_i, & i=L+1,\cdots,2L \end{cases} \quad (1)$$

where h is a scaling parameter and L is the dimension of the state $\hat{x}$. The subscript i indicates the ith column of the matrix. A set of the posterior sigma points can be derived by propagating these prior sigma points through the nonlinear function g: $\mathscr{Z}_i = g(\chi_i)$. Furthermore, the estimations of mean $\hat{z}$, covariance P_z and cross-covariance P_{xz} are obtained as follows:

$$\bar{z} \approx \sum_{i=0}^{2L} w_i^{(m)} \mathscr{Z}_i \quad (2)$$

$$P_z \approx \sum_{i=1}^{L} w_i^{(c_1)} (\mathscr{Z}_i - \mathscr{Z}_{i+L})(\mathscr{Z}_i - \mathscr{Z}_{i+L})^T + \sum_{i=1}^{L} w_i^{(c_2)} (\mathscr{Z}_i + \mathscr{Z}_{i+L} - 2\mathscr{Z}_0)(\mathscr{Z}_i + \mathscr{Z}_{i+L} - 2\mathscr{Z}_0)^T \quad (3)$$

$$P_{xz} \approx \sqrt{w_1^{(c1)} P_x} (\mathscr{Z}_{1:L} - \mathscr{Z}_{L+1:2L})^T \quad (4)$$

The corresponding weights for the mean and covariance are defined as:

$$\begin{aligned} w_0^{(m)} &= \tfrac{h^2-L}{h^2} \\ w_i^{(m)} &= \tfrac{1}{2h^2}, \\ w_i^{(c1)} &= \tfrac{1}{4h^2}, \\ w_i^{(c2)} &= \tfrac{h^2-1}{4h^4}, \quad i=1,\cdots,2L \end{aligned} \quad (5)$$

As proved in [4], if the random variables obey a Gaussian distribution, the optimal value of h is $\sqrt{3}$. Stirling's interpolation only depends on one parameter which is the interval size h in contrast to three parameters (α, β, κ) which are required in unscented transform. This makes Stirling's method simpler and easier adjustable.

3. CENTRAL DIFFERENCE INFORMATION FILTER

In this section, we present our CDIF framework which is developed by embedding Stirling's interpolation into the UIF structure. To achieve this, the sigma points are derived by Stirling's interpolation instead of the unscented transform. The algorithm includes three steps: prediction, measurement update and global information fusion.

3.1. Prediction

Here we consider the discrete-time nonlinear dynamic system:

$$x_{k+1} = f(x_k, w_k), \quad (6)$$

where x_k is the state vector of the system at time step k, and $w_k \sim \mathscr{N}(0, Q_k)$ is the zero mean Gaussian noise.

First, the state vector is augmented with the noise variable and the corresponding augmented covariance matrix is derived by

$$x_k^{a_w} = \begin{bmatrix} x_k \\ w_k \end{bmatrix}, P_k^{a_w} = \begin{bmatrix} P_k & 0 \\ 0 & Q_k \end{bmatrix} \quad (7)$$

A symmetric set of $2L+1$ sigma points is generated by using (1)

$$\chi_{i,k}^{a_w} = \begin{cases} x_k^{a_w}, & i=0 \\ x_k^{a_w} + (h\sqrt{P_x^{a_w}})_i, & i=1,\cdots,L \\ x_k^{a_w} - (h\sqrt{P_x^{a_w}})_i, & i=L+1,\cdots,2L \end{cases} \quad (8)$$

where each sigma point $\chi_{i,k}^{a_w}$ contains the state and noise variable components

$$\chi_{i,k}^{a_w} = \begin{bmatrix} \chi_{i,k}^{x} \\ \chi_{i,k}^{w} \end{bmatrix} \quad (9)$$

These sigma points are further passed through the nonlinear function (6), such that the predicted sigma points for the discrete time $k+1$ are derived

$$\chi_{i,k+1|k}^{x} = f(\chi_{i,k}^{x}, \chi_{i,k}^{w}) \quad (10)$$

Finally, the first two moments of the predicted state vector are obtained by linear regression of the transformed sigma points

$$x_{k+1|k} = \sum_{i=0}^{2L} w_i^m \chi_{i,k+1|k}^{x} \quad (11)$$

$$P_{k+1|k} = \sum_{i=1}^{L} w_i^{(c_1)}(\chi_i - \chi_{i+L})(\chi_i - \chi_{i+L})^T + \sum_{i=1}^{L} w_i^{(c_2)}(\chi_i + \chi_{i+L} - 2\chi_0)(\chi_i + \chi_{i+L} - 2\chi_0)^T \quad (12)$$

As stated in Section 1, the information matrix and information vector are the dual of the mean and covariance, so that the predicted information matrix $Y_{k+1|k}$ and the information vector $y_{k+1|k}$ are derived as:

$$Y_{k+1|k} = (P_{k+1|k})^{-1} \quad (13)$$

$$y_{k+1|k} = Y_{k+1|k} x_{k+1|k} \quad (14)$$

3.2. Measurement update

The measurement function of the nonlinear system is defined as

$$z_k = h(x_k) + v_k, \quad (15)$$

where z_k is the measurement and $v_k \sim \mathcal{N}(0, R_k)$ is the Gaussian noise of the measurement.

The sigma points used for the measurement update are derived as:

$$\chi_{i,k+1|k} = \begin{cases} x_{k+1|k}, & i = 0 \\ x_{k+1|k} + (h\sqrt{P_{k+1|k}})_i, & i = 1, \cdots, L \\ x_{k+1|k} - (h\sqrt{P_{k+1|k}})_i, & i = L+1, \cdots, 2L \end{cases} \quad (16)$$

The predicted measurement points are obtained by transforming the sigma points through (15)

$$\mathscr{Z}_{i,k+1|k} = h(\chi_{i,k+1|k}) \quad (17)$$

Furthermore, the mean and cross-covariance are derived by

$$z_{k+1|k} = \sum_{i=0}^{2L} w_i^m \mathscr{Z}_{i,k+1|k} \quad (18)$$

$$P^{xz}_{k+1|k} = \sqrt{w_1^{(c1)} P_{k+1|k}}(\mathscr{Z}_{1:L} - \mathscr{Z}_{L+1:2L})^T \quad (19)$$

Both Eq. (16) and Eq. (19) require the calculation of $\sqrt{P_{k+1|k}}$, so we only need calculate it once for each time step. Finally, the measurement update of the information vector and the information matrix for sensor are derived as:

$$y_{k+1} = y_{k+1|k} + \phi_{k+1}, \quad (20)$$

$$Y_{k+1} = Y_{k+1|k} + \Phi_{k+1}, \quad (21)$$

where ϕ_{k+1} and Φ_{k+1} are information contributions for the information vector and matrix respectively, which can be derived by

$$\phi_{k+1} = Y_{k+1|k} P^{xz}_{k+1|k} R^{-1}_{k+1}[z_{k+1} - z_{k+1|k} + (P^{xz}_{k+1|k})^T y_{k+1|k}] \quad (22)$$

$$\Phi_{k+1} = Y_{k+1|k} P^{xz}_{k+1|k} R^{-1}_{k+1} (P^{xz}_{k+1|k})^T Y_{k+1|k} \quad (23)$$

The mathematic derivation of Eq. (22) and Eq. (23) can be found in [1, 2].

3.3. Global information fusion

In case of multiple sensors, e.g., N, where the measurement noises are uncorrelated between the sensors, the measurement update for the information fusion is simply expressed as a linear combination of the local information contribution terms by:

$$y_{k+1} = y_{k+1|k} + \sum_{i=1}^{N} \phi_{i,k+1} \quad (24)$$

$$Y_{k+1} = Y_{k+1|k} + \sum_{i=1}^{N} \Phi_{i,k+1} \quad (25)$$

4. SIMULATION EXPERIMENTS

To demonstrate the performance of CDIF, here we consider a classic space vehicle reentry tracking problem, which was used in [1, 6, 7]. Two radars, which measure range and bearing, are used for tracking a high speed vehicle.

The state space of the filter consists of the position (x_1 and x_2), the velocity (x_3 and x_4) and a parameter related to the aerodynamic force x_5. As described in [6], the vehicle state dynamics for the discrete case are given by

$$\begin{aligned} x_1(k+1) &= x_1(k) + \Delta t x_3(k) \\ x_2(k+1) &= x_2(k) + \Delta t x_4(k) \\ x_3(k+1) &= x_3(k) + \Delta t(D(k)x_3(k) + G(k)x_1(k)) + w_1(k) \\ x_4(k+1) &= x_4(k) + \Delta t(D(k)x_4(k) + G(k)x_2(k)) + w_2(k) \\ x_5(k+1) &= x_5(k) + \Delta t w_3(k), \end{aligned} \quad (26)$$

where $w_1(k)$, $w_2(k)$, $w_3(k)$ are Gaussian process noises, $D(k)$ is the drag-related force, $G(k)$ is the gravity-related force, and $\Delta t = 0.1s$ is the sampling time. The force terms are given by

$$\begin{aligned} D(k) &= \beta(k)V(k)exp\left\{\frac{R_0 - R(k)}{H_0}\right\} \\ G(k) &= -\frac{Gm_0}{R^3(k)}, \end{aligned} \quad (27)$$

where $\beta(k) = \beta_0\, exp\{x_5(k)\}$, $R(k) = \sqrt{x_1^2(k) + x_2^2(k)}$ is the distance between the vehicle and the earth center, and $V(k) = \sqrt{x_3^2(k) + x_4^2(k)}$ is the vehicle's speed. The constants in (27) are defined as: $\beta_0 = -0.59783, H_0 = 13.406, Gm_0 = 3.9860 \times 10^5, R_0 = 6374$. The discrete process noise covariance in our simulation is defined by

$$Q(k) = diag(2.4064 \times 10^{-5}, 2.4064 \times 10^{-5}, 10^{-6}), \quad (28)$$

where $diag$ means the diagonal matrix. The vehicle is tracked by radars which are located at (x_s, y_s), where $s = 1, 2$, and the measurement model is

$$\begin{aligned} r_s(k) &= \sqrt{(x_1(k) - x_s)^2 + (x_2(k) - y_s)^2} + e_{r,s}(k) \\ \theta_s(k) &= tan^{-1}\left(\frac{x_2(k) - y_s}{x_1(k) - x_s}\right) + e_{\theta,s}(k), \end{aligned} \quad (29)$$

where $[e_{r,s}(k), e_{\theta,s}(k)]^T \sim \mathcal{N}(0, R_s(k))$ is the measurement noise. In the simulation, the radars are located at $(x_1, y_1) =$

Table 1: Means and standard deviations of RMSE values of the position and average run time in 100 Monte Carlo runs of the reentry tracking problem

Method	E[RMSE]	STD[RMSE]	Average run time(s)
UIFa	0.0083	0.0007	2.0488
CDIFa	0.0083	0.0007	2.0422
UIFb	0.0060	0.0005	2.0575
CDIFb	0.0060	0.0005	2.0496

$(6474, 0)$ and $(x_2, y_2) = (6475, -30)$, and their measurement noise variances are

$$R_1(k) = diag((1 \times 10^{-3})^2, (1.7 \times 10^{-4})^2) \\ R_2(k) = diag((2 \times 10^{-3})^2, (1.7 \times 10^{-4})^2). \quad (30)$$

The initial true state and the covariance of the vehicle are given by

$$x_0 = [6500.4, 349.14, -1.8093, -6.7967, 0.6932]^T \\ P_0 = diag(10^{-6}, 10^{-6}, 10^{-6}, 10^{-6}, 0), \quad (31)$$

and the prior state and the covariance are given by

$$\hat{x}_0 = [6500.4, 349.14, -1.8093, -6.7967, 0]^T \\ \hat{P}_0 = diag(10^{-6}, 10^{-6}, 10^{-6}, 10^{-6}, 1), \quad (32)$$

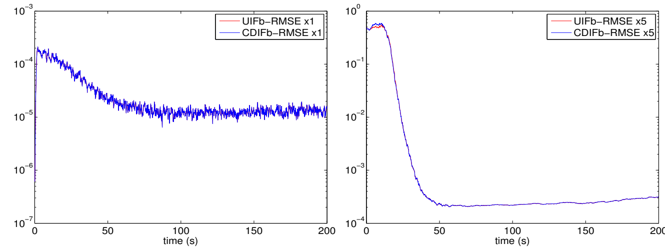
which are same as used in [6].

The time step Δt in the (26) is set to $0.1s$, and measurements from both radars are received during each step, such that the observation frequency of two radars is $10Hz$.

The results of the simulation are derived from 100 Monte Carlo simulations, which are shown in Table 1, where CDIFa and UIFa consider only the measurements from the first radar, and CDIFb and UIFb consider measurements from both radars. The results indicate that by fusing more sensor information the CDIF and UIF can achieve much more accurate results, i.e., the mean and the standard deviation of the root mean square error (RMSE) of the position decrease, whereas the additional computational cost for fusion is very low (0.42% for UIF and 0.36% for CDIF in this simulation). Furthermore, the CDIF runs slightly faster than the UIF in the simulation, although they have almost identical RMSE over time, which is shown in Fig. 1.

5. CONCLUSION

In this paper, a new central difference information filter (CDIF) algorithm for multiple sensor fusion and target tracking was presented. Stirling's interpolation employed in the CDIF only depends on one parameter (interval size) in contrast to three parameters which are required in the unscented transform, which makes the CDIF simpler, faster and easier adjustable than the UIF. A vehicle reentry tracking application is employed to demonstrate the performance of our algorithm. The simulation results show that the new method not only inherits the simplicity and accuracy of the UIF, but also has lower computational cost for multiple sensor fusion.



(a) (b)

Fig. 1: The RMSE error of x_1 and x_5 against the time.

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